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Universal amplitude ratios for self-avoiding walks on the kagome lattice

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Abstract. We have calculated exactly the number, the mean-square end-to-end distance, the mean-square radius of gyration, and the mean-square distance of a monomer from the origin, for n-step self-avoiding walks on the kagome lattice up to 30 steps. We have estimated the connective constant and the critical amplitudes. Our numerical results are consistent with the theoretical prediction by Cardy and Saleur on the universality for certain amplitude ratios.

A self-avoiding walk (SAW) is a model of a polymer (de Gennes 1979). Recently, several combinations of critical amplitudes for two-dimensional SAWs were predicted to be lattice-independent (Cardy and Saleur 1989, Caracciolo *et al* 1990, Cardy and Guttmann 1993). These predictions have been confirmed for SAWs on the square, triangular and honeycomb lattices. The motivation of the present paper is to study numerically SAWs on the kagome lattice, and our results are indeed consistent with universality. Recent progress on SAWs have been reviewed by Lin and Hsaio (1993).

We are mainly interested in the following functions: (i) the chain generating function for SAWs $C(x) = \sum c_n x^n$ where c_n is the total number of n-step SAWs; (ii) the mean-square end-to-end distance of n-step SAWs $\langle R_{\rm e}^2 \rangle_n$; (iii) the mean-square radius of gyration of n-step SAWs $\langle R_{\rm g}^2 \rangle_n$; and (iv) the mean-square distance of a monomer from the origin $\langle R_{\rm m}^2 \rangle_n$ of n-step SAWs.

The asymptotic forms at large n are believed to be (Cardy and Guttmann 1993)

$$c_n \approx A\mu^n n^{\gamma-1}$$
$$\langle R_{\rm e}^2 \rangle_n \approx Bn^{2\nu} \qquad \langle R_{\rm g}^2 \rangle_n \approx Cn^{2\nu} \qquad \langle R_{\rm m}^2 \rangle_n \approx Dn^{2\nu}$$

where μ is the connective constant. The exponents γ and ν depend only on the space dimensionality d and not on the particular lattice chosen. The amplitudes A, B, C, D and the connective constant μ vary from lattice to lattice. Exact values for the exponents have been derived for $d = 2$ and the results are (Nienhuis 1982)

$$\gamma = 43/32 = 1.343\,75 \qquad \nu = 3/4 = 0.75. \tag{1}$$

For $d = 3$ the exact results are not available and we have

$$\gamma \approx 7/6 \qquad \nu \approx 3/5. \tag{2}$$

Although the amplitudes are lattice-dependent, Cardy and Saleur (1989) used the c-theorem in conformal theory to prove that the amplitude ratios C/B and D/B are universal. However, their theoretical predictions are not consistent with numerical results (Guttmann and Yang 1990, Lam 1990). A minor mistake in the work of Cardy and Saleur (1989) was

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Table 1. Exact enumeration results for the number, mean-square end-to-end distance, mean-square radius of gyration and mean-square distance of a monomer from the origin, for self-avoiding walks on the kagome lattice.

n	$c_n/4$	$c_n\langle R_e^2\rangle_n/4$	$(n+1)^2c_n\langle R_g^2\rangle_n/4$	$(n+1)c_n\langle R_m^2\rangle_n/4$
1	1	1	1	1
2	3	8	14	11
3	8	41	111	72
4	22	171	682	371
5	60	640	3 553	1 663
6	163	2 239	16 575	6 798
7	436	7 482	71 051	25 960
8	1 154	24 166	286 049	94 212
9	3 052	76 001	1 099 912	329 151
10	8 082	233 994	4 075 589	1 115 867
11	21 352	708 191	14 619 729	3 688 286
12	56 160	2 113 386	50 989 263	11 930 762
13	147 256	6 232 392	173 732 733	37 902 618
14	385 736	18 193 209	580 602 208	118 601 581
15	1 009 814	52 641 701	1 907 864 402	366 279 322
16	2 640 138	151 149 326	6 174 608 132	1 118 080 962
17	6 891 872	431 066 945	19 713 357 863	3 377 815 789
18	17 969 517	1 222 017 563	62 186 629 663	10 111 620 182
19	46 815 736	3 445 704 922	194 090 889 621	30 024 000 396
20	121 881 736	9 668 955 109	599 972 747 158	88 497 495 657
21	317 067 290	27 013 693 275	1 838 398 101 862	259 118 611 346
22	824 208 073	75 173 755 581	5 588 057 184 927	754 100 047 365
23	2 141 102 780	208 438 089 633	16 861 686 012 443	2 182 498 482 624
24	5 558 956 276	576 033 125 833	50 539 116 310 822	6 284 590 732 291
25	14 425 260 268	1 587 058 080 403	150 545 962 391 624	18 012 489 393 834
26	37 414 334 468	4 360 292 627 183	445 886 809 762 145	51 404 096 550 537
27	96 994 722 794	11 948 322 465 757	1 313 632 063 383 791	146 112 141 517 962
28	251 416 952 773	32 671 205 585 607	3 852 100 833 376 732	413 882 091 090 767
29	651 055 515 786	89 086 229 814 062	11 237 995 271 643 907	1 167 724 197 217 793
30	1 685 795 303 428	242 471 516 931 336	32 653 731 490 314 182	3 284 859 612 821 943

corrected by Caracciolo *et al* (1990). From exact enumeration results for the mean-square distance of the monomers to the origin, and the centre of mass to the origin on the square lattice up to 21 steps and the triangular lattice up to 15 steps, Guttmann and Yang (1990) obtained for both lattices

$$C/B = 0.1396 \pm 0.001 \qquad D/B = 0.4375 \pm 0.002. \tag{3}$$

From a Monte Carlo study of SAWs on the square lattice, Caracciolo *et al* found that

$$C/B = 0.140\,26 \pm 0.000\,11 \qquad D/B = 0.439\,62 \pm 0.000\,33. \tag{4}$$

We have calculated the number, mean-square end-to-end distance, mean-square radius of gyration, and mean-square distance of a monomer from the origin, of SAWs on the kagome lattice up to 30 steps and give the results in table 1. We used the first-order differential approximants (Guttmann 1989) based on the series of c_n to estimate the connective constant μ and the critical amplitude A:

$$\mu = 2.5606 \pm 0.0002 \qquad A = 1.162 \pm 0.001. \tag{5}$$

The amplitude ratios C/B and D/B are calculated by extrapolation techniques (Guttmann 1989). We find

$$\begin{aligned} &B = 0.848 \pm 0.001 \qquad C = 0.119 \pm 0.001 \qquad D = 0.373 \pm 0.001 \\ &C/B = 0.140 \pm 0.001 \qquad D/B = 0.440 \pm 0.001. \end{aligned} \tag{6}$$

Our numerical results are in agreement with the prediction of Cardy and Saleur (1989) on the universality of certain amplitude ratios.

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